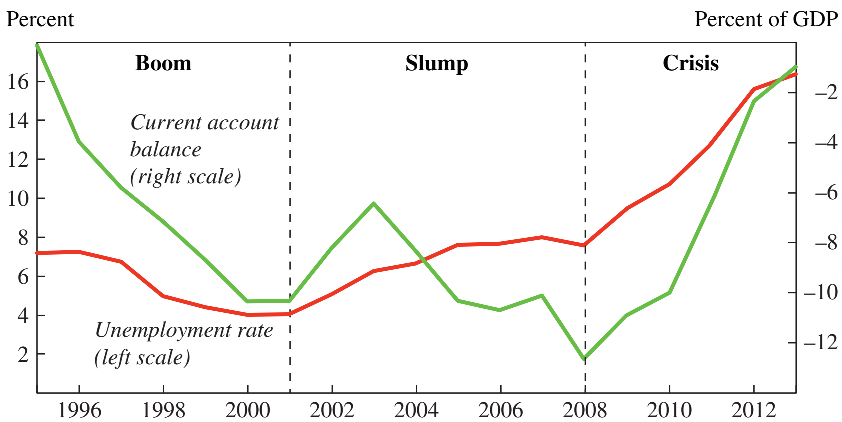
460-2 International Economics

Lecture notes 1: Current account

1 Boom and bust

- Portugal 1995-2013 boom-bust cycle

Figure 1. Current Account Balance and Unemployment Rate in Portugal, 1995–2013

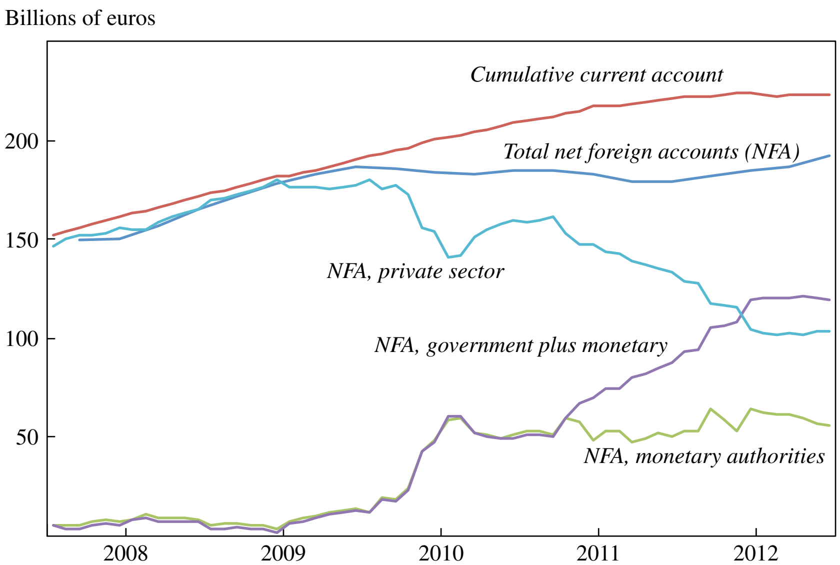


Source: International Monetary Fund, World Economic Outlook database.

- Chapter 1: Boom
- Capital inflows driven by optimistic expectations (sharp decline in interest rate on Portuguese borrowing), boom and real appreciation
- Chapter 2: Slump
- Disappointment, private demand slows down, growth slows down, exports remain weak and Portugal keeps running a current account deficit
- Nontradable sector keeps expanding at expenses of tradable sector
- Wages keep growing despite productivity growing slower than in trading partners
- Chapter 3: Crash
- World financial crisis, drop in exports, low output, non-performing loans, low tax revenue, fiscal trouble

- Sudden stop in financial flows, adjustment in current account, smoothed by official flows (first Target 2 balances, then troika package)
- Adjustment in current account eventually takes place, mostly through severe contraction in output/imports

Figure 6. Private Capital Flows in Portugal, 2008–12



- References:
  - Blanchard, Olivier. 2007. "Adjustment within the Euro: The Difficult Case of Portugal." *Portuguese Economic Journal* 6, no. 1: 1–21.
  - Ricardo Reis. 2013. "The Portuguese Slump and Crash and the Euro Crisis." *Brookings Papers on Economic Activity.* (see also Blanchard's discussion of this paper)
- Questions: what causes of capital flows?
- How do capital flows affect real activity, relative prices (exchange rate)?
- Always good to keep in mind current account identity:

$$\text{Capital account} + \text{current account} = 0$$

so we always need to square what happens in capital market with what happens in goods market

2 The intertemporal approach to the current account

2.1 One good, endowment economy

Consider a small open economy where a single good is produced and consumed. The agents in the economy can borrow or lend at the constant world interest rate r, which they take as given. In other words, there is a single asset traded on international capital markets: a risk-free bond with price $1/(1+r)$. There is a representative consumer with preferences:

$$E\sum_{t=0}^{\infty}\beta^t u(c_t)$$

We start by considering an endowment economy. There is an exogenous income stream $\{y_t\}$, the same for all consumers. The flow budget constraint is:

$$a_t = (1+r)a_{t-1} + y_t - c_t,$$

where a_t is the net asset position of the agent. The consumer has to satisfy a no-Ponzi condition, that is, almost surely:

$$\lim_{T\to\infty}(1+r)^{-T}a_T = 0$$

The world interest rate r is constant. The only source of uncertainty is the endowment process.

Assume that y_t is a Markov process. Later, we will focus on two interesting special cases: the AR1 process

$$y_t = (1-\rho)\bar{y} + \rho y_{t-1} + \epsilon_t \tag{1}$$

with $\rho \in [0,1)$; and the AR2 process with a unit root:

$$\Delta y_t = \rho\Delta y_{t-1} + \epsilon_t. \tag{2}$$

We are interested in characterizing the dynamics of the current account, that is the change in the net foreign asset position of the country

$$CA_t = a_t - a_{t-1}$$

The basic balance of payments identity is:

$$a_t - a_{t-1} = y_t - c_t + ra_{t-1} \tag{3}$$

The term $y_t - c_t$ is the trade balance and ra_{t-1} is net interest payments on international loans.

We will add some assumptions. First, we assume that the utility takes the quadratic form:

$$u(c) = c - \frac{1}{2}bc^2$$

Second, we assume that:

$$\beta(1+r) = 1.$$

This means that borrowing and lending is not the result of differences in time preferences between the country and the rest of the world. Under these assumptions, we'll derive a general formula for optimal consumption—equation (5) below—which holds under different assumptions for the process of y_t. Notation, price of a 1 period bond:

$$q = \frac{1}{1+r}.$$

First notice that an optimal path consumption must satisfy the necessary Euler equation

$$u'(c_t) = \beta(1+r) E_t [u'(c_{t+1})].$$

Given quadratic preferences this yields

$$c_t = E_t [c_{t+1}].$$

Iterating and using the law of iterated expectations we obtain

$$c_t = E_t [c_{t+j}]. \tag{4}$$

Consider the budget constraints at times $t, t+1, ...$

$$\begin{aligned} a_t &= (1+r) a_{t-1} + y_t - c_t \\ a_{t+1} &= (1+r) a_t + y_{t+1} - c_{t+1} \\ &\ldots \end{aligned}$$

Sum side by side, discounting by $(1+r)^{-j}$ the j-th equation. Using the no Ponzi condition this yields the intertemporal budget constraint:

$$\sum_{j=0}^{\infty} (1+r)^{-j} (y_{t+j} - c_{t+j}) + (1+r) a_{t-1} = 0.$$

Taking expectations $E_t [.]$ on both sides and using (4), we get

$$\sum (1+r)^{-j} (E_t y_{t+j} - c_t) + (1+r) a_{t-1} = 0,$$

which, rearranged, gives the general consumption equation we were looking for

$$c_t = \frac{r}{1+r} \left[\sum (1+r)^{-j} E_t [y_{t+j}] + (1+r) a_{t-1} \right]. \tag{5}$$

The interpretation is that it is optimal to consume just the interest on your total wealth, which is the sum of your financial wealth a_{t-1} plus your human wealth $\sum (1+r)^{-j} E_t [y_{t+j}]$. For example, in the special case of the AR1 (1) we obtain

$$c_t = \frac{1-\rho}{1+r-\rho} \bar{y} + \frac{r}{1+r-\rho} y_t + r a_{t-1},$$

so consumption responds positively to an income shock ε_t and the response is larger as $\rho \to 1$.

Let me derive a useful result. For any sequence $\{x_t\}$ with well defined discounted sum, the following holds

$$\begin{aligned}
(1-\beta)\sum_{j=0}^{\infty}\beta^j x_{t+j} &= (1-\beta)\,x_t + (1-\beta)\,\beta x_{t+1} + (1-\beta)\,\beta^2 x_{t+2} + ... \\
&= x_t + \beta\,(x_{t+1}-x_t) + \beta^2\,(x_{t+2}-x_{t+1}) + ... \\
&= x_t + \sum_{j=1}^{\infty}\beta^j \Delta x_{t+j}.
\end{aligned}$$

Applying this result to (5) we obtain

$$\begin{aligned}
c_t &= \frac{r}{1+r}\left[\sum (1+r)^{-j} E_t\,[y_{t+j}] + (1+r)\,a_{t-1}\right] \\
&= y_t + \sum_{j=1}^{\infty}\beta^j E_t\,[\Delta y_{t+j}] + r a_{t-1}.
\end{aligned}$$

2.2 Time series implications

We can now derive the model implications for the current account:

$$CA_t = a_t - a_{t-1} = y_t + r a_{t-1} - c_t \tag{6}$$

$$= -\sum_{j=1}^{\infty}\beta^j E_t\,[\Delta y_{t+j}] \tag{7}$$

Interpretation: the current account forecasts future decreases in income. You accumulate assets in times in which you think your permanent income is going to decline.

This suggests empirical tests, which have been developed by Sheffrin and Woo (1990). To do so, we must reinterpret the model and identify y_t with output net of investment and of government spending $(Y - I - G)$. The idea is that conditional on the path for Y, I and G, consumers' optimality implies the conditions above. In other words they are necessary conditions that also hold when we add capital accumulation and government spending.

Test 1: GMM Minimal assumptions on income process. Test the condition:

$$E_{t-1}\,[CA_{t-1} - \beta CA_t + \beta \Delta y_t] = 0 \tag{8}$$

To get it take E_{t-1} on equation (7):

$$E_{t-1}\,[CA_t] = -\sum_{j=1}^{\infty}\beta^j E_{t-1}\,[\Delta y_{t+j}]$$

and use the lagged CA equation

$$\begin{aligned} CA_{t-1} &= -\sum_{j=1}^{\infty} \beta^j E_{t-1}\left[\Delta y_{t-1+j}\right] = -\beta E_{t-1}\left[\Delta y_t\right] - \beta \sum_{j=1}^{\infty} \beta^j E_{t-1}\left[\Delta y_{t+j}\right] \\ &= -\beta E_{t-1}\left[\Delta y_t\right] + \beta E_{t-1}\left[CA_t\right] \end{aligned}$$

Using (8), we can run a test of orthogonality using any variable X_{t-1} known at $t-1$:

$$E_{t-1}\left[\left(CA_{t-1} - \beta CA_t + \beta \Delta y_t\right) X_{t-1}\right] = 0,$$

and taking unconditional expectations yields

$$E\left[\left(CA_{t-1} - \beta CA_t + \beta \Delta y_t\right) X_{t-1}\right] = 0.$$

This suggests that we look at the sample analog

$$\frac{1}{T} \sum_{t=1}^{T} \left(CA_{t-1} - \beta CA_t + \beta \Delta y_t\right) X_{t-1} \rightarrow 0$$

whose properties are known from GMM.

2.2.1 Test 2: Restriction on VAR coefficients

Making more assumptions on the income process, we obtain tighter testable implications. In particular, assume that the joint dynamics of CA_t and Δy_t are well captured by a bivariate VAR with J lags:

$$\left[\begin{array}{c} CA_t \\ \Delta y_t \end{array}\right] = \sum_{j=1}^{J} \Psi_j \left[\begin{array}{c} CA_{t-j} \\ \Delta y_{t-j} \end{array}\right] + e_t.$$

Write it as

$$Z_t = \Psi Z_{t-1} + e_t$$

where Z_t includes all the lags used

$$Z_t = \left[\begin{array}{c} CA_t \\ \Delta y_t \\ CA_{t-1} \\ \Delta y_{t-1} \\ \ldots \end{array}\right]$$

Then

$$E_t Z_{t+j} = \Psi^j Z_t$$

and the expressions in equation (7) can be written as follows

$$CA_t = \left[\begin{array}{cccc} 1 & 0 & 0 & \ldots \end{array}\right] Z_t,$$

just from the definition of Z_t, and

$$\begin{aligned}\sum_{j=1}^{\infty}\beta^j E_t\left[\Delta y_{t+j}\right] &= \sum\beta^j \begin{bmatrix} 0 & 1 & 0 & ... \end{bmatrix} E_t Z_{t+j} \\ &= \begin{bmatrix} 0 & 1 & 0 & ... \end{bmatrix} \sum\beta^j \Psi^j Z_t.\end{aligned}$$

Then given the estimated parameters in Ψ we can test whether

$$\begin{bmatrix} 0 & 1 & 0 & ... \end{bmatrix} \beta\Psi\left(I-\beta\Psi\right)^{-1} = \begin{bmatrix} 1 & 0 & 0 & ... \end{bmatrix}.$$

2.2.2 Forecasting

Just form expectations in (7):

$$\widehat{CA}_t = \begin{bmatrix} 0 & 1 & 0 & ... \end{bmatrix} \beta\Psi\left(I-\beta\Psi\right)^{-1} Z_t.$$

Then we can also do out-of-sample forecasting (i.e. estimate Ψ using data up to time t, and compare $\widehat{CA}_{t+j}$ and CA_{t+j} for $j = 1, 2, ...$).

2.3 Growth shocks

General idea: response of savings, investment and CA to shocks depends on future time profile of income.

2.3.1 AR1 in levels

Suppose income process is

$$y_t = (1-\rho)\,\bar{y} + \rho y_{t-1} + \varepsilon_t.$$

Then, using (5) we get

$$c_t = \bar{y} + \frac{r}{1+r-\rho}\left(y_t - \bar{y}\right) + r a_{t-1}$$

and

$$CA_t \quad = y_t - c_t + r a_{t-1} = \quad \frac{1-\rho}{1+r-\rho}\left(y_t - \bar{y}\right).$$

Two implications:

1. Positive correlation between CA and income. Booms lead to CA surplus, recessions to CA deficit.
2. Consumption is less volatile than income.

To derive the second implication notice that

$$Var_{t-1}[c_t] = \left(\frac{r}{1+r-\rho}\right)^2 Var_{t-1}[y_t]$$

so

$$\frac{Var_{t-1}[c_t]}{Var_{t-1}[y_t]} = \left(\frac{r}{1+r-\rho}\right)^2 < 1$$

since $\rho < 1$.

2.3.2 AR1 in growth rates

Suppose income process is

$$\Delta y_t = \rho \Delta y_{t-1} + \varepsilon_t.$$

Using (7) we have

$$CA_t = -\sum_{j=1}^{\infty} q^j E_t[\Delta y_{t+j}] = -\sum_{j=1}^{\infty} q^j \rho^j \Delta y_t = -\frac{q\rho}{1-q\rho}\Delta y_t$$

and

$$c_t = y_t - CA_t + ra_{t-1} = y_t - \frac{q\rho}{1-q\rho}\Delta y_t + ra_{t-1}.$$

Two implications:

1. Negative correlation between CA and income growth. Positive current shock implies growing income profile, leads to borrowing in anticipation of higher income in the future.
2. Consumption is more volatile than income.

To derive the second implication notice that

$$\begin{aligned} Var_{t-1}[c_t] &= \left(\frac{1}{1-q\rho}\right)^2 \sigma_\varepsilon^2, \\ Var_{t-1}[y_t] &= \sigma_\varepsilon^2. \end{aligned}$$

So

$$\frac{Var_{t-1}[c_t]}{Var_{t-1}[y_t]} = \left(\frac{1}{1-q\rho}\right)^2 > 1.$$

The observation that the relative volatility of consumption and income depend on the income process goes back to Deaton (1987) work on "excess smoothness" of consumption.

Aguiar and Gopinath use the implications above idea to explain volatility in emerging economies. They have a real business cycle model in which income is driven by TFP and in which the process for TFP features both shocks to levels (as in our first model) and shocks to growth rates (as in our second model). They then argue that the role of the two shocks is different in developed vs emerging economies.

- developed economies: more transitory shocks→
  - low ratio $Var[\Delta c] / Var[\Delta y]$
  - $Corr[CA, \Delta y]$ negative but low (in abs value)
- emerging economies: more permanent shocks→
  - high ratio $Var[\Delta c] / Var[\Delta y]$
  - $Corr[CA, \Delta y]$ negative and large (in abs value)

3 Current account and the real exchange rate

- Let's add relative prices
- Tradable and non-tradable goods

$$c_t = \left(c_t^T\right)^{\alpha} \left(c_t^N\right)^{1-\alpha}$$

- Production functions of tradables and non-tradables

$$\begin{aligned} y_t^T &= A_t^T f\left(n_t^T\right) \\ y_t^N &= A_t^N f\left(n_t^N\right) \end{aligned}$$

- Specific factors so f are strictly concave
- Inelastic supply of labor equal to 1, so production possibility frontier is

$$f^{-1}\left(y_t^T / A_t^T\right) + f^{-1}\left(y_t^N / A_t^N\right) = 1$$

- Budget constraint is now

$$a_t = (1+r)a_{t-1} + w_t + \Pi_t - c_t^T - p_t c_t^N$$

  (using tradable as numeraire)

- Optimality conditions for household

$$\begin{aligned} \alpha u'(c) \frac{c}{c^T} &= \lambda \\ (1-\alpha) u'(c) \frac{c}{c^N} &= p\lambda \end{aligned}$$

  so

$$\frac{1-\alpha}{\alpha} \frac{c^T}{c^N} = p$$

  and Euler equation

$$u'(c_t) \frac{c_t}{c_t^T} = \beta(1+r) E_t \left[u'(c_{t+1}) \frac{c_{t+1}}{c_{t+1}^T} \right]$$

- Optimality condition for firms

$$\begin{aligned} A^T f'(n^T) &= w \\ pA^N f'(n^T) &= w \end{aligned}$$

- Market clearing in non-tradables

$$c_t^N = y_t^N$$

- Allocation between tradables and non tradables

$$\frac{1-\alpha}{\alpha}\frac{c^T}{c^N} = p = \frac{A^T f'(n^T)}{A^N f'(n^N)}$$

so we have a decreasing relation between c^T and n^T

$$\frac{1-\alpha}{\alpha}c^T = A^T \frac{f'(n^T)}{f'(1-n^T)} f(1-n^T)$$

and from this we can derive a decreasing relation between c^T and y^T

- Wages plus profits equal value of total output so

$$a_t = (1+r)a_{t-1} + y_t^T + p_t y_t^N - c_t^T - p_t c_t^N$$

using market clearing in NT

$$a_t = (1+r)a_{t-1} + y_t^T - c_t^T$$

- In general we need to solve jointly for T and NT allocation
- Special case: $u(c) = \log c$ then we can first solve for intertemporal allocation of T and derive NT allocation in second step
- Euler equation becomes

$$\frac{1}{c_t^T} = \beta(1+r)E_t\left[\frac{1}{c_{t+1}^T}\right]$$

- Assuming

$$\beta(1+r) = 1$$

- Suppose constant levels of A_t^j and $a_{t-1} = 0$, no shocks
- At date 0 there is a one time, permanent unexpected shock to A_t^T
- Optimal to set c_t^T and y_t^T constant and equal to each other both before and after the shock
- So unexpected permanent shock to A^T

  - c^T increases proportionally to A^T
  - c^N unchanged
  - p increases

- Consider now anticipated shock: at t expect A_t^T to be higher starting at $t+1$
- Increase in c^T, c^N first goes up, then down, p appreciates and then appreciates more

3.1 Real exchange rate

- Tradable, non-tradable, home, foreign goods

$$c = \left(c^T\right)^{\alpha}\left(c^N\right)^{1-\alpha}$$

$$c^T = \left(c^H\right)^{\omega}\left(c^F\right)^{1-\omega}$$

- Maximization involves always

$$\max_{c^H, c^F, c^N} C\left(c^H, c^F, c^N\right)$$

$$p^H c^H + p^F c^F + p^N c^N \leq X$$

where C is an aggregator homogeneous of degree 1 so solution yields

$$C = \frac{X}{P}$$

where P is a price index, function of p^H, p^F, p^N and homogeneous of degree 1

- In case of Cobb-Douglas

$$P = \left(\left(p^H\right)^{\omega}\left(p^F\right)^{1-\omega}\right)^{\alpha}\left(p^N\right)^{1-\alpha}$$

(omitting multiplicative constant)

- If foreign good denominated in foreign currency and nominal exchange rate is e we have

$$P = \left(\left(p^H\right)^{\omega}\left(ep^{F*}\right)^{1-\omega}\right)^{\alpha}\left(p^N\right)^{1-\alpha}$$

- Price index for foreign consumers is

$$P^* = \left(\left(p^{F*}\right)^{\omega}\left(p^H/e\right)^{1-\omega}\right)^{\alpha}\left(p^{N*}\right)^{1-\alpha}$$

- Real exchange rate is

$$\frac{eP^*}{P} = \frac{\left(\left(ep^{F*}\right)^{\omega}\left(p^H\right)^{1-\omega}\right)^{\alpha}\left(ep^{N*}\right)^{1-\alpha}}{\left(\left(p^H\right)^{\omega}\left(ep^{F*}\right)^{1-\omega}\right)^{\alpha}\left(p^N\right)^{1-\alpha}} = \left(\frac{ep^{F*}}{p^H}\right)^{\alpha(2\omega-1)}\left(\frac{ep^{N*}}{p^N}\right)^{1-\alpha}$$

  driven by two forces:

  - a term of trade component ep^{F*}/p^H
  - a relative price of non-tradables ep^{N*}/p^N

- Real exchange rate and real interest rate

- Euler equation can always be derived as

$$u'(c_t) = \beta(1+r_t)E\left[\frac{P_t}{P_{t+1}}u'(c_{t+1})\right]$$

- Changes in relative prices mean that the term $(1+r_t)P_t/P_{t+1}$ is different for different countries (even in real models)

4 The transfer problem

- Now consider a model with only tradable goods, but differentiated by country

- Consumption is aggregate of home good c_{ht} and foreign good c_{ft}

$$c_t = \xi c_{ht}^{\omega} c_{ft}^{1-\omega}$$

  with $\xi = \omega^{-\omega}(1-\omega)^{-(1-\omega)}$

- Budget constraint is now

$$a_t = p_{ht}y_{ht} - p_{ht}c_{ht} - p_{ft}c_{ft} + (1+r_t)a_{t-1}$$

- Consumer optimality implies that the domestic demand for the home good is

$$c_{ht} = \omega\left(\frac{p_{ht}}{p_t}\right)^{-1}c_t, \tag{9}$$

  where the domestic consumer price index p_t includes the price of the home and foreign good and is

$$p_t = p_{ht}^{\omega}p_{ft}^{1-\omega} \tag{10}$$

- Consider now the case of two symmetric countries

- The demand of home goods by foreign country is

$$c^*_{ht} = (1-\omega)\left(\frac{p_{ht}}{p^*_t}\right)^{-1} c^*_t \tag{11}$$

- Consider endowment economy
- We now ask how the relative price

$$\frac{p_{ht}}{p_{ft}}$$

(the domestic terms of trade) is related to the current account surplus

$$\Delta = (1+r)a_{t-1} - a_t$$

- Rewrite the budget constraint as

$$pc = p_h y_h + \Delta$$

- Equilibrium in the market for the home good is then

$$\omega\left(\frac{p_h}{p}\right)^{-1}\left(\frac{p_h y_h + \Delta}{p}\right) + (1-\omega)\left(\frac{p_h}{p^*}\right)^{-1}\left(\frac{p_f y_f - \Delta}{p^*}\right) = y_h$$

or

$$\omega\frac{p_h y_h + \Delta}{p_h} + (1-\omega)\frac{p_f y_f - \Delta}{p_h} = y_h$$

- Use foreign good as numeraire $p_f = 1$ what is the effect of a financial transfer Δ on the relative price p_h?
- If $\omega = 1/2$ the effect is zero
- Home bias in consumption $\omega > 1/2$ implies

$$\frac{dp_h}{d\Delta} > 0$$

5 The Portfolio Approach to the Current Account

- A model based on Kraay and Ventura
- Consumers can invest in three assets: international bonds, home capital and foreign capital
- Total investment in the three assets is denoted by b_t, k_t, k^*_t
- Home capital and foreign capital are risky with random i.i.d. linear returns A_t and A^*_t

- No labor income
- World interest rate constant at r
- Consumers preferences are represented by

$$E\left[\sum_{t=0}^{\infty}\beta^t \ln c_t\right]$$

- Flow budget constraint is

$$b_{t+1}+k_{t+1}+k_{t+1}^{*}+c_t=A_t k_t+A_t^{*}k_t^{*}+(1+r)\,b_t$$

- Define wealth as

$$w_t=A_t k_t+A_t^{*}k_t^{*}+(1+r)b_t$$

- Optimality conditions

$$\begin{aligned}\frac{1}{c_t} &= \beta E_t\frac{A_{t+1}}{c_{t+1}}\\ \frac{1}{c_t} &= \beta E_t\frac{A_{t+1}^{*}}{c_{t+1}}\\ \frac{1}{c_t} &= \beta(1+r)E_t\frac{1}{c_{t+1}}\end{aligned}$$

- Define ratios

$$\begin{aligned}\theta_{t+1} &= \frac{k_{t+1}}{w_t-c_t}\\ \theta_{t+1}^{*} &= \frac{k_{t+1}^{*}}{w_t-c_t}\end{aligned}$$

so

$$b_t=(1-\theta_t-\theta_t^{*})(w_t-c_t)$$

- Sum term by term optimality conditions after multiplying by ratios

$$\begin{aligned}\frac{1}{c_t} &= \beta E_t\frac{\theta_t A_{t+1}+\theta_t^{*}A_{t+1}^{*}+(1-\theta_t-\theta_t^{*})(1+r)}{c_{t+1}}=\\ &= \beta E_t\frac{1}{c_{t+1}}\frac{w_{t+1}}{w_t-c_t}\end{aligned}$$

given that

$$\theta_t A_{t+1}+\theta_t^{*}A_{t+1}^{*}+(1-\theta_t-\theta_t^{*})(1+r)=\frac{w_{t+1}}{w_t-c_t}$$

is the rate of return on the country's portfolio

- Conjecture consumption to wealth ratio is constant, then

$$\frac{w}{c} - 1 = \beta\left(\frac{w}{c}\right)$$

gives $c_t = (1-\beta)w_t$

- The optimal portfolio share then come from

$$\frac{w_t - c_t}{c_t} = \beta E_t \frac{w_{t+1}}{c_{t+1}} \frac{A_{t+1}}{\theta_t A_{t+1} + \theta_t^* A_{t+1}^* + (1-\theta_t - \theta_t^*)(1+r)}$$

and similar equation for k^*, which yield two non-linear equations in θ, θ^*

$$\begin{aligned} 1 &= E\left[\frac{A}{\theta A + \theta^* A^* + (1-\theta-\theta^*)(1+r)}\right] \\ 1 &= E\left[\frac{A^*}{\theta A + \theta^* A^* + (1-\theta-\theta^*)(1+r)}\right] \end{aligned}$$

- So optimal policy is

$$\begin{aligned} c_t &= (1-\beta)\, w_t \\ k_{t+1} &= \theta\beta w_t \\ k_{t+1}^* &= \theta^*\beta w_t \\ b_{t+1} &= (1-\theta-\theta^*)\,\beta w_t \end{aligned}$$

- Suppose foreign consumers save β of their wealth and invest a fraction $\hat{\theta}$ in domestic assets

- Current account implications

$$CA_t = b_{t+1} - b_t + k_{t+1}^* - k_t^* - (k_{t+1}^f - k_t^f) = (1-\theta)\,\beta\,(w_{t+1} - w_t) + \hat{\theta}\beta(w_{t+1}^f - w_t^f)$$

National savings

$$\begin{aligned} NS_t &= (A_t - 1)\, k_t + (A_t^* - 1)\, k_t^* + rb_t - c_t \\ &= k_{t+1} - k_t + b_{t+1} - b_t + k_{t+1}^* - k_t^* = \\ &= \beta\,(w_{t+1} - w_t) \end{aligned}$$

- Rest of the world savings $WS_t = \beta(w_{t+1}^f - w_t^f)$

$$CA_t = (1-\theta)\, NS_t + \hat{\theta} WS_t$$

- Since

$$1 - \theta = \frac{k_t^* + b_t}{k_t + k_t^* + b_t} = \frac{\text{foreign assets}}{\text{total assets}} = X_t$$

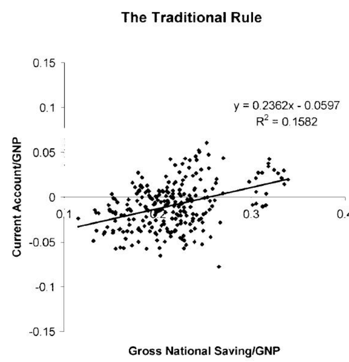
we have the implication

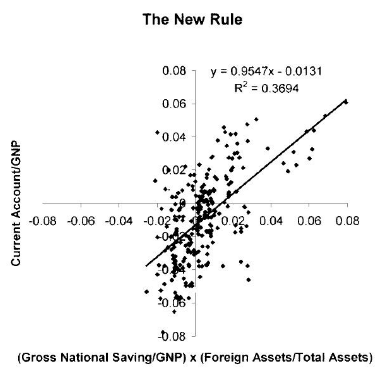
$$CA_t = b \cdot X_t \cdot NS_t + \varepsilon_t$$

(orthogonality of ε?)

- Run this regression and see if coefficient b is close to 1
- Straw man "the traditional rule": regression

$$CA_t = b \cdot NS_t + \varepsilon_t$$





- Careful, we are running regression

$$NFA_{t+1} - NFA_t = b \cdot \frac{FA_t}{TA_t} \cdot (TA_{t+1} - TA_t) + \varepsilon_t$$

or

$$FA_{t+1} - FA_t - \Delta HA^*_{t+1} = b \cdot \left(FA_t \frac{TA_{t+1}}{TA_t} - FA_t \right) + \varepsilon_t$$

so spurious correlation is a concern

- General relation (this is an identity):

$$\begin{aligned} CA_t &= \frac{FA_t}{TA_t}(TA_{t+1} - TA_t) + \left(\frac{FA_{t+1}}{TA_{t+1}} - \frac{FA_t}{TA_t} \right) TA_{t+1} - \\ & \quad \frac{HA_t}{WA_t}(WA_{t+1} - WA_t) + \left(\frac{HA_{t+1}}{WA_{t+1}} - \frac{HA_t}{WA_t} \right) WA_{t+1} \end{aligned}$$

- How much of variation in CA_{t+1} is due to components 1 and 3 how much to 2 and 4 (portfolio shifts)?

5.1 Accounting and valuation effects with asset prices

- In the model above valuation effects are not present as the price of capital is constant at 1
- We now do some simple manipulations on the budget constraint of a model in which the household sector holds securities whose price is changing
- Domestic household sector, holds a portfolio of J stocks

$$\sum_j p_{jt} s_{jt+1} + c_t = y_t + \sum_j (p_{jt} + d_{jt}) s_{jt}$$

where y_t is labor income

- Stocks are divided in two subsets H home and F foreign so we can rewrite the budget constraint as

$$\begin{aligned} &\sum_{j\in H} p_{jt} \left(S_{jt+1} - s^*_{jt+1}\right) + \sum_{j\in F} p_{jt} s_{jt+1} + c_t = \\ =&y_t + \sum_{j\in H} (p_{jt} + d_{jt}) \left(S_{jt} - s^*_{jt}\right) + \sum_{j\in F} (p_{jt} + d_{jt}) s_{jt} \end{aligned}$$

where S is the total supply of the stock and s^* are foreign holdings

- The country net financial assets are

$$NFA_t = \sum_{j\in F} p_{jt} s_{jt+1} - \sum_{j\in H} p_{jt} s^*_{jt+1}$$

- The current account is then

$$CA_t = NFA_t - NFA_{t-1} = \sum_{j\in F} p_{jt} s_{jt+1} - \sum_{j\in H} p_{jt} s^*_{jt+1} - \left(\sum_{j\in F} p_{jt-1} s_{jt} - \sum_{j\in H} p_{jt-1} s^*_{jt}\right)$$

and the budget constraint can be written as

$$\begin{aligned} \sum_{j\in H} p_{jt} S_{jt+1} + NFA_t + c_t &= y_t + \sum_{j\in H} (p_{jt} + d_{jt}) S_{jt} + NFA_{t-1} + \\ &+ \sum_{j\in F} \frac{p_{jt} + d_{jt} - p_{jt-1}}{p_{jt-1}} p_{jt-1} s_{jt} - \sum_{j\in H} \frac{p_{jt} + d_{jt} - p_{jt-1}}{p_{jt-1}} p_{jt-1} s^*_{jt} \end{aligned} \tag{12}$$

- Notice that the following is the flow from the business sector to the household sector

$$\sum_{j\in H} (p_{jt} + d_{jt}) S_{jt} - \sum_{j\in H} p_{jt} S_{jt+1}$$

which corresponds to financial income net of investment

- So
$$y_t + \sum_{j \in H} (p_{jt} + d_{jt}) S_{jt} - \sum_{j \in H} p_{jt} S_{jt+1} - c_t = Y_t - c_t - i_t$$
domestic savings minus investment

- Then the budget constraint (12) can be rewritten as

$$\begin{aligned} CA_t &= Y_t - c_t - i_t + \\ &+ \sum_{j \in F} \frac{p_{jt} + d_{jt} - p_{jt-1}}{p_{jt-1}} p_{jt-1} s_{jt} - \sum_{j \in H} \frac{p_{jt} + d_{jt} - p_{jt-1}}{p_{jt-1}} p_{jt-1} s^*_{jt} \end{aligned}$$

- The last two terms of this equation capture the return on the country net financial position

- When people refer to "valuation effects" they refer to the change in prices that show up in this last term. Since asset prices are all expressed in terms of domestic currency valuation effects also include the effects of exchange rate fluctuations on asset values